APPM 2350
Exam 1 Solutions

Problem 1

a) Find $\vec{V}(t)$.

$$\vec{V}(t) = |\vec{V}| \frac{\vec{V}}{|\vec{V}|} = |\vec{V}| T = \boxed{-\sin(t)\,\hat{\imath} + 6\hat{\jmath} + \cos t\,\hat{k}}$$

b) $$N = \frac{dT/dt}{|dT/dt|} = \frac{1/\sqrt{37}\left(-\cos(t)\,\hat{\imath} - \sin t\,\hat{k}\right)}{1/\sqrt{37}} = \boxed{-\cos t\,\hat{\imath} - \sin t\,\hat{k}}$$

c) $B = T \times N$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{1}{\sqrt{37}}\sin(t) & 6/\sqrt{37} & \frac{\cos t}{\sqrt{37}} \\ -\cos t & 0 & -\sin t \end{vmatrix} = \boxed{\left\langle \frac{6}{\sqrt{37}}\sin t, \frac{1}{\sqrt{37}}, \frac{-6}{\sqrt{37}}\cos t \right\rangle}$$

d) $t^* = 2\pi$

$\vec{r}(t) = \int V(t) = \langle \cos t + c_1,\ 6t + c_2,\ \sin t + c_3 \rangle$

Solve for C's using $\vec{r}(2\pi) = (1, 12\pi, 0)$

$\Rightarrow C_1 = C_2 = C_3 = 0$

So $\vec{r}(t) = \langle \cos t, 6t, \sin t \rangle$

e) $r(2\pi) = \frac{a}{\sqrt{37}}\langle 0, 6, 1\rangle + b\langle -1, 0, 0\rangle + c\langle 0, \frac{1}{\sqrt{37}}, -6\rangle$

*note $\vec{r}(t) = \langle \cos t, 6t, \sin t \rangle \Rightarrow r(2\pi) = \langle 1, 12\pi, 0\rangle$

so

$\frac{a}{\sqrt{37}}(0) - b + c(0) = 1 \Rightarrow$ $b = -1$

$\frac{a}{\sqrt{37}}(6) + b(0) + \frac{c}{\sqrt{37}} = 12\pi$ } $\Rightarrow$ $c = -12\pi/\sqrt{37}$

$\frac{a}{\sqrt{37}}(1) + b(0) + c(-6) = 0$ } $\Rightarrow$ $a = \frac{72\pi}{\sqrt{37}}$

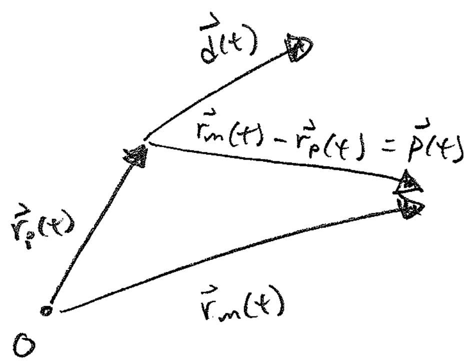
2. a) $\vec{r}_p(t) = \int \vec{v}(t)\,dt = \left(\frac{8}{3}\cos 3t + c_1\right)\hat{i} + (6t + c_2)\hat{j} + \left(\frac{8}{3}\sin 3t + c_3\right)\hat{k}$

$$\vec{r}_p\left(\tfrac{\pi}{2}\right) = c_1\hat{i} + (3\pi + c_2)\hat{j} + \left(-\tfrac{8}{3} + c_3\right)\hat{k} = 0\hat{i} + 3\pi\hat{j} - \tfrac{8}{3}\hat{k}$$

$$\Rightarrow \quad c_1 = c_2 = c_3 = 0$$

$$\boxed{\vec{r}_p(t) = \tfrac{8}{3}\cos 3t\,\hat{i} + 6t\,\hat{j} + \tfrac{8}{3}\sin 3t}$$

b)



Communicates when $\vec{d}(t)$ is parallel to $\vec{p}(t)$.

$$\vec{p}(t) \times \vec{d}(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3t^2}{\pi} - \frac{8}{3}\cos 3t & 0 & \tan 3t - \frac{8}{3}\sin 3t \\ -\cos 3t & 0 & -\sin 3t \end{vmatrix} = \vec{0}$$

$$= \underset{\cdots}{\text{algebra}} = \hat{j}\left(\frac{3t^2}{\pi} - 1\right)\sin 3t = \vec{0}$$

$$\Rightarrow \quad \left(\frac{3t^2}{\pi} - 1\right)\sin 3t = 0$$

$$\frac{3t^2}{\pi} = 1$$

$$\boxed{t = \sqrt{\tfrac{\pi}{3}}}$$

c) $\sin 3t = 0 \quad \Rightarrow \quad 3t = n\pi$

$$\boxed{t = \frac{n\pi}{3}} \qquad n = 1, 2, \ldots$$

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3) a) $\vec{D}_1 = (5,10,10) \approx (1,2,2)$

b) $\vec{D}_2 = (-10,-10,-10) = (-1,-1,-1)$

c) $$\vec{D}_3 = \frac{-\vec{D}_1}{|\vec{D}_1|} + \frac{\vec{D}_2}{|\vec{D}_2|}$$
$$= \frac{(-1,-2,-2)}{3} + \frac{(-1,-1,-1)}{\sqrt{3}}$$
$$= \frac{1}{3}\left[(-1,-2,-2) + (-\sqrt{3},-\sqrt{3},-\sqrt{3})\right]$$
$$= \frac{1}{3}\left[(-1-\sqrt{3},-2-\sqrt{3},-2-\sqrt{3})\right]$$

d) Set $A = \frac{-1-\sqrt{3}}{3}$ & $B = \frac{-2-\sqrt{3}}{3}$. Then
$$A(x-10) + B(y-10) + B(z-10) = 0$$

e) $$-D_1 \cdot D_2 = |D_1||D_2| \cos\theta$$
$$(1+2+2) = 3\sqrt{3} \cos\theta$$
$$\Rightarrow \cos\theta = \frac{5}{3\sqrt{3}} = \frac{5\sqrt{3}}{9}$$

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a) 3

b) 1

c) 4, 7

d) 5

e) 2, 8